Lecture 10: The Physics of Neutron Moderation

CBE 30235: Introduction to Nuclear Engineering — D. T. Leighton

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1 Introduction: The Fork in the Road (Fast vs. Thermal)

In Lecture 9, we defined the cross section (σ). In this lecture we shall show that for Uranium-235, the probability of fission is massive at low energies ($\sim$580 barns at 0.025 eV) but tiny at high energies ($\sim$1 barn at 2 MeV).

When a fission event occurs, the new neutrons are born **Fast** ($E \approx 2$ MeV). This presents the nuclear engineer with two fundamental choices:

1.1 Option A: The Thermal Reactor (The focus of this course)

We deliberately slow the neutrons down using a **Moderator** (water, graphite) to take advantage of the massive low-energy cross section.

- **Pros:** We can use natural or slightly enriched uranium (3-5%). The physics is easier to control.
- **Cons:** We lose neutrons to absorption during the slowing-down process.
- **Examples:** LWR (USA), CANDU (Canada), HTGR.

1.2 Option B: The Fast Reactor

We keep the neutrons moving at high speeds (no moderator) to fission ^{238}U or breed Plutonium.

- **Pros:** At high energies, the neutron yield per fission (ν) is higher, allowing for "Breeding" (creating more fuel than is consumed). This allows us to close the fuel cycle (as done in France/Russia).
- **Cons:** Since the cross section is so small ($\sim$1 barn), the fuel must be highly enriched ($> 20\%$) to sustain a chain reaction. The coolant cannot contain hydrogen (usually liquid Sodium or Lead).

Decision: For the next several weeks, we will focus exclusively on **Option A**. To understand Thermal Reactors, we must understand the mechanics of how a neutron loses energy. This process is called **Moderation**.

2 The Life Cycle of a Neutron (Part 1)

In Lecture 9, we defined the probability of interaction (the cross section, σ). Today, we look at the consequences of those interactions. Our goal in a thermal nuclear reactor is to take a "fast" neutron (born at high energy from fission) and slow it down to "thermal" energies where it can easily cause more fission.

This involves distinct physics problems:

1. **Moderation:** How does a neutron lose energy when it hits a nucleus? (Scattering mechanics).
2. **Probability of Scattering:** How likely is a collision at different energies?
3. **Absorption (Losses):** How do we avoid the "traps" (resonances) while slowing down?

3 Dynamics of Neutron Scattering (Moderation)

Primary Reference: Lamarsh, 4th Ed., Section 3.5

To achieve the "Thermal Reactor" described in Option A, we must reduce the neutron energy from 2 MeV to 0.025 eV. This is a factor of 10^8 reduction.

3.1 Energy Loss in a Single Elastic Collision

Consider a neutron (mass 1) hitting a target nucleus (mass A) in an elastic "billiard ball" collision. Conservation of momentum and energy dictates how much energy is transferred.

Visualizing the Physics: The "Head-On" Collision ($A = 1$)

The mechanics of moderation are best understood by looking at the specific case of a neutron hitting a Hydrogen nucleus ($A = 1$) head-on. The easiest way to solve this is to shift our perspective to the **Center of Mass (COM)** frame of reference.

1. **Lab Frame (Before):** The neutron moves at velocity v. The proton is stationary. The "average" velocity of the system (Center of Mass) is $v_{cm} = v/2$.
2. **COM Frame:** We move along with the center of mass.
   - The neutron appears to move right at $v - v/2 = +v/2$.
   - The proton appears to move left at $0 - v/2 = -v/2$.
3. **The Collision:** Since momentum must be zero in the COM frame and energy is conserved, the particles simply bounce off with the same speed but opposite direction.
   - Neutron rebound velocity (COM) $= -v/2$.
4. **Lab Frame (After):** We shift back to the lab frame by adding v_{cm} to the result.
   - Neutron Final Velocity $= (-v/2)_{rebound} + (+v/2)_{frame} = \mathbf{0}$.

Conclusion: In a head-on collision with Hydrogen, the neutron transfers **100%** of its momentum and energy to the proton, stopping dead in its tracks.

Generalizing to Mass A

For heavier nuclei ($A > 1$), the center of mass moves much slower than $v/2$, so the neutron can never lose all its energy. The maximum possible energy loss is determined by the **Collision Parameter**, α:

$$\alpha = \left(\frac{A-1}{A+1}\right)^2 \tag{1}$$

The neutron's energy after one collision will fall somewhere in the range:

$$\alpha E_{in} \leq E' \leq E_{in} \tag{2}$$

Note:

- **Hydrogen ($A = 1$):** $\alpha = 0$. The neutron can lose up to 100% of its energy (as derived above).
- **Lead ($A = 208$):** $\alpha \approx 0.98$. The neutron retains at least 98% of its energy even in a perfect head-on collision.

3.2 Average Logarithmic Energy Decrement (ξ)

Since neutrons scatter at random angles, we care about the *average* behavior. We define ξ (xi) as the average loss in the natural log of energy per collision:

$$\xi = \ln\left(\frac{E_{in}}{E_{out}}\right)_{average} \approx \frac{2}{A + 2/3} \quad \text{(for } A > 1\text{)} \tag{3}$$

This parameter dictates our choice of moderator:

- **Water (H):** $\xi = 1.0$. Requires $\sim$18 collisions to thermalize.
- **Graphite (C):** $\xi = 0.158$. Requires $\sim$115 collisions.
- **Uranium (U):** $\xi = 0.008$. Requires $\sim$2170 collisions.

3.3 Energy Dependence of Elastic Scattering

So far, we have discussed *how much* energy is lost if a collision occurs. But how *likely* is the collision?

For most good moderators (like Hydrogen, Carbon, and Oxygen) in the energy range of interest (1 eV to 100 keV), the elastic scattering cross-section (σ_s) is relatively constant.

- This is often called **Potential Scattering**.
- The neutron essentially sees the nucleus as a hard sphere with cross-section $\sigma_s \approx 4\pi R^2$, where R is the nuclear radius.
- **Significance:** Unlike absorption (which varies wildly, see Section 4), the scattering probability is reliable. This creates a steady "staircase" of energy loss, allowing the neutron to reliably march down from MeV energies toward thermal energies.

3.4 Inelastic Scattering: The "Heavy Lifter" at High Energy

You might ask: "Doesn't inelastic scattering lose even more energy?" Yes. In an inelastic collision ($n + A \to n' + A^*$), the neutron gives up a massive amount of kinetic energy to excite the nucleus.

- **Significance:** This mechanism is very effective for **Heavy Nuclei** (like Uranium or Iron) at high energies (> 1 MeV).
- **The Limitation:** It is a **Threshold Reaction**. It can only occur if the neutron has enough energy to reach the first excited state of the nucleus.
  - For U-238, the threshold is low (~ 45 keV).
  - For Oxygen-16 (moderator), the threshold is huge (~ 6 MeV).

Conclusion: Since fission neutrons are born at ~ 2 MeV, they *cannot* inelastically scatter against the Oxygen in the water. We must rely on **Elastic Scattering** to take the neutron the rest of the way down.

4 Neutron Absorption (Losses)

Primary Reference: Lamarsh Section 3.6

Now that we know *how* the neutron slows down (Section 3), we must examine the treacherous path it must travel. While we want the neutron to scatter, we desperately want to avoid it being **Captured** (Absorbed) before it reaches thermal energy.

4.1 The "Valley of Death" (Resonance Region)

Between 1 eV and 10 keV, the capture cross-section of ^{238}U exhibits a "forest" of sharp, jagged peaks.

- **The Physics:** These peaks correspond to discrete quantum energy levels of the excited compound nucleus (^{239}U*).
- **The Trap:** In ^{238}U, these are almost exclusively **Capture Resonances** (n, γ). If a neutron slows down and lands in one of these energy "traps" (e.g., the massive resonance at 6.67 eV), it is absorbed. It dies without causing fission.
- **The Name:** We call this the "Valley of Death" because neutrons must successfully "jump" over these peaks via elastic scattering to reach the safe thermal region.

4.2 Visualizing the Resonances (Interactive Tool)

Since we cannot see this behavior in a static textbook plot, we will use an online nuclear data plotter. *Note: The standard NNDC site is often offline.*

Primary Tool: KAERI Table of Nuclides https://atom.kaeri.re.kr/nuchart/

Exercise for Students:

1. Go to the URL above.
2. In the "Nuclide" search box, type: `U238` (and hit Enter).

3. **Choose the Library:** Locate the column labeled **ENDF/B-VIII.0** (this is the US standard) and click on the small **+** sign to expand the options.
4. **Select the Data:** In the expanded list, locate the row labeled **Capture cross sections** and click the button labeled **Plot**.
5. **Verify Scale:** The graph that appears is the (n, γ) radiative capture cross section. The axes should default to **Log-Log** scale. (If they appear linear, use the settings to switch them to Log).
6. **Observation:** Look at the energy range between 10^0 eV and 10^4 eV. You will see the massive peaks rising 1000x above the baseline. These are the "Resonance Traps" our moderator design must defeat.

4.3 Region 3: The 1/v Region (Thermal)

Once the neutron passes the last resonance (at roughly 6 eV), it enters the "safe" thermal region. Here, the absorption cross-section rises smoothly according to the **1/v Law**:

$$\sigma_a \propto \frac{1}{v} \propto \frac{1}{\sqrt{E}} \tag{4}$$

This massive rise at low energy is what makes thermal reactors possible.

5 Summary: The Goal of Lecture 10

We have successfully described the journey of the neutron from birth (high energy) to thermal maturity (low energy).

- We defined the **Moderator** (H, C) and why we use it (ξ).
- We analyzed the **Mechanics** of energy loss (α, COM frame).
- We identified the **Enemy**: Resonance Capture in U-238.

However, we have not yet discussed the most important event: **Death and Rebirth**. What happens when the thermal neutron finally hits a ^{235}U nucleus?

In **Lecture 11 (Friday)**, we will cover the **Physics of Fission**, the Liquid Drop Model, and the origin of the massive energy release that powers the reactor.